# Quantum Formulas Collection

←Quantum

**Attribution**: this resource was created by Harold Foppele.

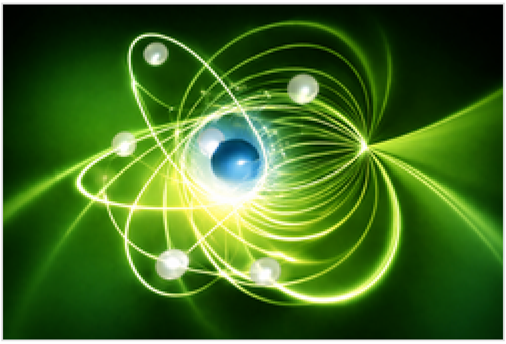
**Subject classification**: this is a physics resource.

**Type classification**: this is a quiz resource.

**Type classification**: this resource is a learning project.

# Introduction

This page provides a list of the most important formulas in quantum mechanics, useful as a quick reference for students, teachers, and researchers. The formulas are organized by topic and include names, mathematical expressions, and short explanations of what they mean and how they are used. While this collection focuses on key results, science is always evolving, and new discoveries may override or extend these formulas. You, the reader, are welcome to suggest additions or corrections to keep this resource up to date.



Artistic impression of an atom 4

### Key Formulas in Quantum Mechanics

This table lists key formulas in quantum mechanics, showing their names, expressions, and applications.

QuantumWiki

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|---|---|---|---|
| Compton Effect: Change in Wavelength | $\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$ | Shift in photon wavelength after scattering. | Compton scattering, evidence for photon momentum. |
| Cutoff Wavelength | $\lambda_{min} = \frac{hc}{K_0}$ | Minimum wavelength in bremsstrahlung. | X-ray production. |
| De Broglie Wavelength | $p = \frac{h}{\lambda} = \hbar k$ | Wavelength associated with a particle's momentum. | Matter waves, electron diffraction. |
| Occupancy Probability | $P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$ | Fermi-Dirac distribution. | Electron statistics in metals. |
| Density of States | $N(E) = 8\sqrt{2}\pi m^{3/2} E^{1/2} / h^3$ | Number of states per energy interval (3D free electron gas). | Solid-state physics, Fermi gas. |
| Dirac Equation | $\left(\beta mc^2 + c\sum_{k=1}^{3}\alpha_k p_k\right)\Psi = i\hbar\frac{\partial}{\partial t}\Psi$ | Relativistic quantum equation for fermions. | Particle physics, electrons. |
| Electric Dipole Potential Energy | $V = -\mathbf{p}\cdot\mathbf{E}$ | Energy of dipole in electric field. | Molecular physics. |
| Electrostatic, Coulomb Potential Energy | $V = \frac{q_1 q_2}{4\pi\epsilon_0 r}$ | Coulomb potential. | Atomic interactions. |
| Free Particle Schrödinger's Equation (1D) | $-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\Psi = E\Psi$ | For free particle in 1D. | Free particle motion. |
| Free Particle Schrödinger's Equation (3D) | $-\frac{\hbar^2}{2m}\nabla^2\Psi = E\Psi$ | For free particle in 3D. | Scattering problems. |
| Harmonic Oscillator Potential Energy | $V = \frac{1}{2}kx^2$ | Potential for harmonic oscillator. | Vibrational modes, quantum optics. |
| Heisenberg's Uncertainty Principle | $\Delta x \Delta p_x \geq \frac{\hbar}{2}$ <br> $\Delta E \Delta t \geq \frac{\hbar}{2}$ | Limits on simultaneous knowledge of position/momentum and energy/time. | Fundamental limit in measurements, quantum tunneling. |
| Hydrogen Atom, Orbital Energy | $E_n = -\frac{me^4}{8\epsilon_0^2 h^2 n^2} = -\frac{13.6\,\mathrm{eV}}{n^2}$ | Energy levels of hydrogen atom. | Atomic spectroscopy, Bohr model. |
| Hydrogen Atom, Radial Probability Density | $P(r) = \frac{4r^2}{a^3}e^{-2r/a}$ | Probability density for electron position (ground state). | Atomic orbitals. |
| Hydrogen Atom Spectrum, Rydberg Equation | $\frac{1}{\lambda} = R_H\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$ | Wavelengths of spectral lines. | Hydrogen emission/absorption spectra. |
| Infinite Potential Well Energy Levels | $E_n = \left(\frac{hn}{2L}\right)^2\frac{1}{2m}$ | Energy levels for particle in a box. | Quantum confinement, nanostructures. |

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| Probability Conservation for Quantum Mechanics | $\frac{\partial}{\partial t}\int_V \lvert\Psi\rvert^2 \, \mathrm{d}V + \int_S \mathbf{j}\cdot \mathrm{d}\mathbf{A} = 0$ | Conservation of probability. | Quantum dynamics. |
| Magnetic Dipole Potential Energy | $V = -\mathbf{m}\cdot\mathbf{B}$ | Energy of dipole in magnetic field. | Magnetic resonance. |
| Moseley's Law | $f = \frac{c}{\lambda} = M_{K_\alpha}(Z-1)^2$ $M_{K_\alpha} = 2.47\times 10^{15}$ Hz | Frequency of K-α X-ray line. | Atomic number determination, X-ray spectroscopy. |
| Normalization Integral | $\int_{\mathbf{r}\in R} \lvert\Psi\rvert^2 \, \mathrm{d}V = 1$ | Normalizes the wavefunction. | Probability calculations. |
| One-Dimensional Box Potential Energy | $V = \begin{cases} 0 & x \in [a,b] \\ \infty & x \notin [a,b] \end{cases}$ | Potential for particle in a box. | Quantum wells. |
| Orbital Electron Magnetic Dipole Components | $\mu_{orb,z} = -m_\ell \mu_B$ | Z-component of orbital magnetic moment. | Zeeman effect. |
| Orbital Electron Magnetic Dipole Moment | $\mu_{orb} = -e\mathbf{L}/2m$ | Magnetic moment due to orbital motion. | Atomic magnetism. |
| Orbital, Electron Magnetic Dipole Moment Potential | $U = -\mu_{orb}\cdot\mathbf{B}_{ext} = -\mu_{orb,z}B_{ext}$ | Potential in external field. | Magnetic interactions. |
| Spin, Electron Magnetic Dipole Moment | $\mu_{\mathbf{s}} = -\frac{e}{m}\mathbf{S} = -g\frac{e}{2m}\mathbf{S}$ | Spin magnetic moment. | Electron spin resonance. |
| Photoelectric Effect: Maximum Kinetic Energy | $E_{\mathrm{k\,max}} = hf - \Phi$ | Maximum kinetic energy of photoelectrons. | Photoelectric effect experiments, solar cells. |
| Photon Momentum | $p = \frac{hf}{c} = \frac{h}{\lambda}$ | Momentum of a photon. | Quantum optics, Compton scattering. |
| Planck–Einstein Equation | $E = hf = \frac{hc}{\lambda}$ | Relates energy of a photon to its frequency or wavelength. | Wave-particle duality, photon energy calculations. |
| Planck's Radiation Law (Frequency Form) | $I(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$ | Spectral radiance for blackbody in frequency. | Blackbody radiation, stellar spectra. |
| Planck's Radiation Law (Wavelength Form) | $I(\lambda,T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$ | Spectral radiance for blackbody in wavelength. | Thermal radiation analysis. |
| Probability Current (Non-Relativistic) | $\mathbf{j} = \frac{\hbar}{2mi}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)$ | Flow of probability. | Current in quantum systems. |
| Probability Density Function | $\rho(\mathbf{r},t) = \lvert\Psi(\mathbf{r},t)\rvert^2$ | Probability density. | Locating particles. |

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|---|---|---|---|
| Momentum Magnitude | $S = \hbar\sqrt{s(s+1)}$ | Magnitude of spin. | Particle spin properties. |
| Spin Projection Quantum Number | $m_s \in \left\{-\frac{1}{2}, +\frac{1}{2}\right\}$ | Spin along z-axis for electrons. | Spintronics, NMR. |
| Time-Dependent Schrödinger's Equation (1D) | $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right)\Psi = i\hbar\frac{\partial}{\partial t}\Psi$ | Time evolution in 1D. | Dynamics of quantum systems. |
| Time-Dependent Schrödinger's Equation (3D) | $\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi = i\hbar\frac{\partial}{\partial t}\Psi$ | Time evolution in 3D. | Quantum simulations. |
| Time-Independent Schrödinger's Equation (1D) | $\left(-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V\right)\Psi = E\Psi$ | Stationary states in 1D. | Bound states, potentials. |
| Time-Independent Schrödinger's Equation (3D) | $\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi = E\Psi$ | Stationary states in 3D. | Atomic and molecular physics. |
| Wavefunction of a Trapped Particle, One Dimensional Box | $\Psi_n(x) = A\sin\left(\frac{n\pi x}{L}\right)$ | Wavefunction for particle in a box. | Bound states, quantum wells. |
| Work Function | $\Phi = hf_0$ | Minimum energy to eject an electron. | Photoelectric effect, surface physics. |

## 2. ORGANIZED BY TOPIC

Below are the same formulas grouped

# QUANTUM MECHANICS (QM)

## Core Dynamical Equations

Time-Dependent Schrödinger Equation $i\hbar, \partial_t \Psi = \hat{H}\Psi$

Time-Independent Schrödinger Equation $\hat{H}\psi = E\psi$

Time-Evolution Operator $U(t) = e^{-i\hat{H}t/\hbar}$

## Operators and Measurement Theory

Canonical Commutation Relation (Heisenberg) $[x, p] = i\hbar$

Expectation Value $\langle A\rangle = \langle\psi|A|\psi\rangle$

## Harmonic Oscillator

Annihilation Operator $a = \frac{1}{\sqrt{2\hbar m\omega}}, (m\omega x + ip)$

Energy Levels $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$

### Perturbation Theory & Quantum Transitions

First-Order Energy Correction $E_n^{(1)} = \langle n|V|n\rangle$

Fermi Golden Rule (Transition Rate) $\Gamma = \frac{2\pi}{\hbar}, |V_{fi}|^2, \rho(E)$

### Continuity Equation & Probability Current

Probability Current $j = \frac{\hbar}{2mi}(\psi^{\nabla}\psi - \psi\nabla\psi^{)}$

# OPEN QUANTUM SYSTEMS

- Density Matrix (Statistical Mixture) $\rho = \sum_i p_i, |\psi_i\rangle\langle\psi_i|$
- Lindblad Master Equation (Markovian Open Systems) $\dot{\rho} = -\frac{i}{\hbar}[\hat{H}, \rho] + \sum_k \mathcal{D}[L_k]\rho$
- von Neumann Entropy $S = -\mathrm{Tr}(\rho \log \rho)$

# QUANTUM INFORMATION SCIENCE (QIS)

$I(A : B) = S(A) + S(B) - S(AB)$

$\Phi(\rho) = \sum_k A_k \rho A_k^\dagger$ (quantum channels)

- Bell states $|\psi^\pm\rangle$, $|\phi^\pm\rangle$
- CNOT gate definition
- Qubit superposition $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$a, a^\dagger$ creation–annihilation operators

$H_{\text{int}} = -\mathbf{d} \cdot \mathbf{E}$

- Coherent state $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$
- Jaynes–Cummings Hamiltonian $H = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_0\sigma_z + g(a^\dagger\sigma_- + a\sigma_+)$

# QUANTUM STATISTICAL MECHANICS

- Partition function $Z = \text{Tr}(e^{-\beta H})$
- Thermal state $\rho_\beta = e^{-\beta H}/Z$
- Response function $\chi(\omega)$

# QUANTUM FIELD THEORY (QFT)

- Canonical commutation $[\phi(x), \pi(y)] = i\hbar\delta(x-y)$
- Klein–Gordon equation $(\Box + m^2)\phi = 0$
- Dirac Lagrangian
- Relativistic dispersion $E^2 = p^2c^2 + m^2c^4$

# 3. MULTI-COLUMN VERSION

- $i\hbar\partial_t\Psi = H\Psi$
- $H\psi = E\psi$
- $\Delta x\, \Delta p \geq \hbar/2$
- $[x, p] = i\hbar$
- $P(a) = |\langle a|\psi\rangle|^2$
- $\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$
- $S = -\text{Tr}(\rho \log \rho)$
- $E_n = \hbar\omega(n + 1/2)$
- $a = (m\omega x + ip)/\sqrt{2\hbar m\omega}$
- $\Gamma = \frac{2\pi}{\hbar}|V_{fi}|^2\rho(E)$
- $I(A:B) = S(A) + S(B) - S(AB)$
- Bell states $|\psi^\pm\rangle$
- CNOT $= |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$
- $\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \sum \mathcal{D}[L]\rho$
- $[\phi(x), \pi(y)] = i\hbar\delta(x-y)$
- $(\Box + m^2)\phi = 0$
- $Z = \text{Tr}(e^{-\beta H})$

# 4. Wave Packet spreading example

Free particle dispersion: $\sigma_x(t) = \sigma_x(0)\sqrt{1 + \left(\frac{\hbar t}{2m\sigma_x(0)^2}\right)^2}$ → Used in cold-atom clouds, ultrafast electron microscopy.

## Harmonic oscillator example

Ground state energy: $E_0 = \frac{1}{2}\hbar\omega$ → Zero-point fluctuations in quantum optics.

| | | |
|---|---|---|
| $\hat{H}\psi = E\psi$ | Time-independent Schrödinger equation | Spectra, tunneling, bound states |
| $\Delta x\, \Delta p \geq \frac{\hbar}{2}$ | Heisenberg uncertainty | Measurement limits, wave packets |
| $[x, p] = i\hbar$ | Canonical commutator | Quantization, oscillators |
| $\langle A \rangle = \langle \psi \vert A \vert \psi \rangle$ | Expectation value | Predictions, statistics |
| $P(a) = \vert \langle a \vert \psi \rangle \vert^2$ | Born rule | Measurement probabilities |
| $\hat{U}(t) = e^{-iHt/\hbar}$ | Time-evolution operator | Quantum gates, scattering |
| $\rho = \sum_i p_i \vert \psi_i \rangle \langle \psi_i \vert$ | Density matrix | Decoherence, open systems |
| $S = -\mathrm{Tr}(\rho \log \rho)$ | von Neumann entropy | Entanglement, thermodynamics |
| $\mathrm{Tr}(\rho A)$ | Expectation via density matrix | Ensembles, thermal states |
| $\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \sum_k \mathcal{D}[L_k]\rho$ | Lindblad master eq. | Decoherence, dissipation |
| $\mathcal{D}[L]\rho = L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}$ | Dissipator | Relaxation, noise |
| $Z = \mathrm{Tr}(e^{-\beta H})$ | Partition function | Thermodynamics, blackbody |
| $\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int dp\, e^{ipx/\hbar} \phi(p)$ | Fourier relation | Wavepackets, scattering |
| $j = \frac{\hbar}{2mi}(\psi^* \nabla \psi - \psi \nabla \psi^*)$ | Probability current | Continuity, tunneling |
| $\hat{a} = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$ | Annihilation operator | QHO, quantum optics |
| $E_n = \hbar\omega(n + \frac{1}{2})$ | HO spectrum | Phonons, cavities |
| $\phi_n(x) = \ldots$ | HO eigenfunctions | Basis for perturbation theory |
| $\hat{H}_{\mathrm{spin}} = -\gamma \mathbf{B} \cdot \mathbf{S}$ | Spin Hamiltonian | NMR, ESR, qubits |
| $\chi(\omega) = \int_0^\infty dt\, e^{i\omega t} C(t)$ | Response function | Conductivity, noise |
| $k = \sqrt{2mE}/\hbar$ | Free-particle wavenumber | Beams, dispersion |
| $\psi(x) = \sum_n c_n \phi_n(x)$ | Basis expansion | Computation, spectral theory |
| $H = H_0 + \lambda V$ | Perturbation theory split | Approximations, resonances |
| $E_n^{(1)} = \langle n \vert V \vert n \rangle$ | 1st-order energy shift | Stark, Zeeman effects |
| $\Gamma = \frac{2\pi}{\hbar} \vert \langle f \vert V \vert i \rangle \vert^2 \rho(E_f)$ | Fermi golden rule | Transition rates |
| $(a \vert b) = \mathrm{Tr}(a^\dagger b)$ | Hilbert-Schmidt inner product | Superoperators, channels |

| | | |
|---|---|---|
| $\lvert\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(\lvert 01\rangle \pm \lvert 10\rangle)$ | Bell states | Entanglement, teleportation |
| $U_{\mathrm{CNOT}} = \lvert 0\rangle\langle 0\rvert \otimes I + \lvert 1\rangle\langle 1\rvert \otimes X$ | CNOT gate | Quantum computing |
| $[\phi(x), \pi(y)] = i\hbar\delta(x-y)$ | Canonical QFT commutator | Field quantization |
| $E^2 = p^2c^2 + m^2c^4$ | Relativistic dispersion | QFT, particles |
| $\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$ | Dirac Lagrangian | Fermions, QED |
| $\Box\phi + m^2\phi = 0$ | Klein-Gordon eq. | Bosons, relativistic waves |

## See also

- Quantum
- Quantum A Matter Of Size
- Quantum A Spooky Action at a Distance
- Quantum: A Walk Through the Universe
- Number of independent spatial modes in a spherical volume
- Quantum Computing Algorithms in the NISQ Era
- Quantum Formulas Collection
- Quantum Matter Elements and Particles
- Quantum mechanics
- Quantum mechanics/Timeline
- Quantum mechanics measurements
- Quantum Noisy Qubits
- Quantum optics beam splitter experiments
- Quantum: The Secret of Cohesion: How Waves Hold Matter Together
- Quantum Ultra fast lasers
- Template:Quantum optics operators
- Physical Sciences

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