QuantumWiki

Quantum Noisy Qubits

←Quantum

Introduction

This resource is intended for advanced undergraduate or graduate learners in physics or quantum information science. It assumes familiarity with linear algebra, quantum mechanics, and density matrix formalism. The page serves as a self-study and research-oriented overview of noise models in quantum systems.

Attribution: this resource was created by Harold Foppele.

Subject classification: this is a physics resource.

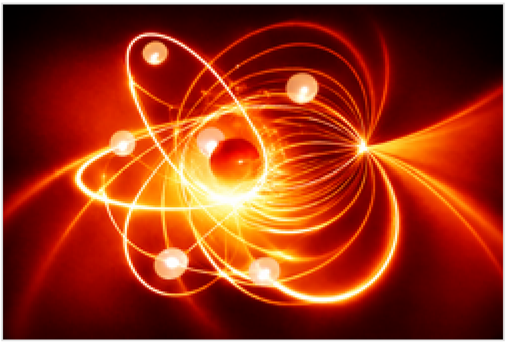
Type classification: this is a quiz resource.

Type classification: this resource is a learning project.

Formulas for Noisy Qubits

Open-system formulations

Field	Quantum computing Qubit physical implementation
Applications	Noise modeling, Decoherence, Error correction
Related topics	Open quantum system Quantum decoherence

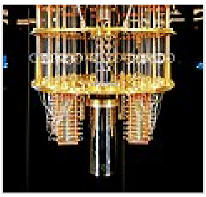


Artistic impression of an atom 7

Noisy qubits are a fundamental challenge in current Noisy Intermediate-Scale Quantum (NISQ) computers, where physical qubits are susceptible to errors from decoherence, are sensitive to their environment (noisy), imperfect gate operations, and measurement noise. These errors stem from interactions with the environment and can accumulate during computations, limiting the depth and complexity of algorithms that can be successfully run. Quantum advantage by quantum processors containing up to 1,000 qubits.[1] Researchers are developing NISQ algorithms that leverage limited resources within these noise constraints and exploring new quantum materials and qubit designs to create more robust qubits for the future of fault-tolerant quantum computing.[2] How qubits interact with their surrounding environment. Unlike isolated quantum systems, real qubits are affected by noise sources such as stray photons, phonons, or control hardware fluctuations. These interactions cause errors including decoherence[3] and relaxation that degrade computational performance.

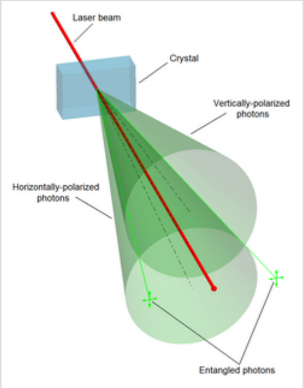
Open system models provide mathematical tools for analyzing and mitigating these effects.[4]
IBM's 50-qubit quantum computer prototype, as exhibited at CES 2018 in Las Vegas ----->
They describe how methods from the theory of Open quantum system are applied to qubits and quantum hardware. In practice, qubits are never perfectly isolated: they interact with their environments, leading to

Textbooks and surveys



50-Qubit

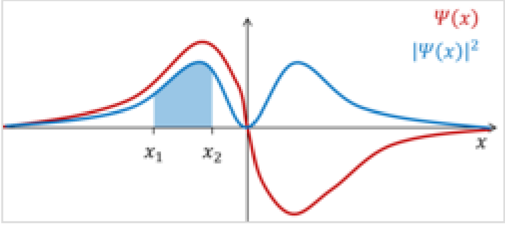
treat this intersection as a distinct domain: Breuer & Petruccione's The Theory of Open Quantum Systems (2002) and Rivas & Huelga's Open Quantum Systems: An Introduction (2012) present explicit applications to quantum information. Reviews such as Krantz et al., A quantum engineer's guide to superconducting qubits (2019), and Preskill, Quantum Computing in the NISQ Era and beyond (2018), emphasize that open-system models underpin both noise characterization and the definition of the NISQ regime. Recent tutorials, e.g. Li et al. (2023), treat simulation of open-system dynamics as a computational task in its own right.



Conceptual illustration of entanglement

As a result, open-system formulations have become central in analyzing qubit performance, setting error-correction thresholds, and guiding fault-tolerant architectures.

Background formulas that govern noisy qubits



Schrödinger-equation example

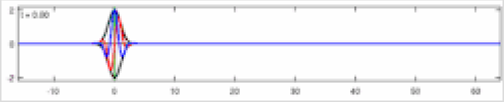
Equation used in wave mechanics (see Quantum mechanics) for the wave function of a particle is the time-independent Schrödinger equation

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-U)\psi = 0$$

It can also be written in operator form as:

$$H\psi = E\psi$$

where ψ is the wave function, ∇^2 the Laplace operator, h the Planck constant, m the particle's mass, E its total energy, and U its potential energy. It was devised by Erwin Schrödinger, who was mainly responsible for wave mechanics.



Schrödinger equation wave packet

The time-dependent Schrödinger equation (https://phys.libretexts.org/Bookshelves/Nuclear_and_Particle_Physics/Introduction_to_Applied_Nuclear_Physics_(Cappellaro)/06%3A_Time_Evolution_in_Quantum_Mechanics/6.01%3A_Time-dependent_Schrodinger_equation#:~:text=Unitary%20Evolution,same%20information:) (see also Dyson series) for an isolated system is: $i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$.

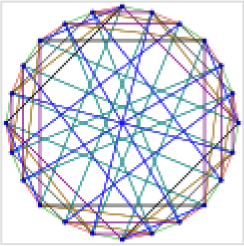
The unitary propagator is:

For open systems, the state of the system alone is obtained from the full density matrix of system+environment:

$$\rho_S(t) = \mathrm{Tr}_E\,[\rho_{SE}(t)].$$

This partial trace generally produces non-unitary dynamics.

From microscopic models to master equations

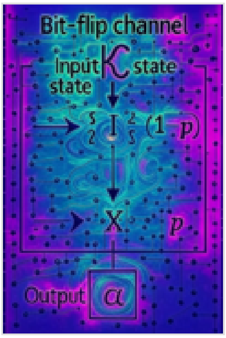

Hamiltonian

Consider a system Hamiltonian (quantum mechanics) H_S, environment Hamiltonian H_E, and an interaction H_I. The total Hamiltonian is:

$$H = H_S + H_E + H_I.$$

Even if ρ_{SE} evolves unitarily, the reduced density matrix ρ_S typically obeys an integro-differential equation. Approximations lead to different master equations.

Kraus representation

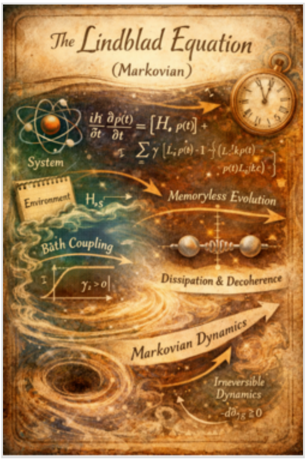


Any completely positive trace-preserving (CPTP) map on a quantum state can be written as:

$$\mathcal{E}(\rho) = \sum_k E_k\,\rho\,E_k^\dagger, \qquad \sum_k E_k^\dagger E_k = I,$$

where the E_k are Kraus operators. This is a general representation of open-system dynamics at discrete times.[5] The bit-flip channel is a quantum channel that, with probability p, applies a qubit flip (X-gate), and with probability $1-p$ does nothing. It is regarded as the quantum analogue of the noise through entanglement with the environment.

Lindblad equation (Markovian)



Lindblad equation (Markovian)

Under the Born–Markov approximation (weak coupling and short environment correlation times), the system's density matrix satisfies the Lindblad master equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \tfrac{1}{2}\{L_k^\dagger L_k, \rho\} \right).$$

This generator defines a dynamical semigroup (completely positive, trace-preserving evolution).[6]

For a single qubit, collapse operators commonly model:

- **Relaxation (energy decay):** $L_{\text{relax}} = \sqrt{\gamma_1}\,\sigma_-$
- **Dephasing:** $L_{\text{deph}} = \sqrt{\gamma_\phi}\,\sigma_z$

Here γ_1 and γ_ϕ are the relaxation and dephasing rates, respectively.

$$\frac{d\rho_S(t)}{dt} = -\frac{i}{\hbar}[H_S, \rho_S(t)] - \int_0^t d\tau \, \mathrm{Tr}_E\big([H_I(t), [H_I(\tau), \rho_S(\tau) \otimes \rho_E]]\big).$$

Redfield theory can describe structured environments (e.g. spin baths or photonic reservoirs) but does not guarantee complete positivity without further corrections.[7]

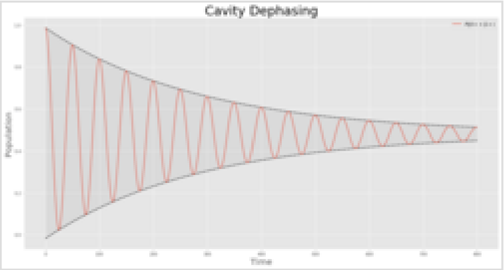
Redfield equation (non-Markovian)

Collisional decoherence

Spin-exchange collisions between alkali metal atoms can change the hyperfine state of the atoms while preserving total angular momentum of the colliding pair. As a result, spin-exchange collisions cause decoherence There has been significant work on correctly identifying the pointer states in the case of a massive particle decohered by collisions with a fluid environment, A widely used approximation for collisional decoherence assumes exponential suppression of off-diagonal terms:

$$C(t) \approx \exp(-\Gamma t), \qquad \Gamma \propto n \, v \, \sigma_{\text{decoh}},$$

with n the particle density, v the relative velocity, and σ_{decoh} the scattering cross-section.[8]



Cavity loses coherence due to dephasing

Applications in quantum mechanics

Open-system formulations are essential in quantum hardware design and analysis:

- **Noise modeling:** Estimating dephasing and relaxation times (T_1, T_2) in superconducting qubits and trapped ions.[9]
- **Error correction:** Providing physical noise models for the design of error-correcting codes.
- **Control techniques:** Informing pulse-shaping and dynamical decoupling sequences to suppress decoherence.
- **Fault tolerance:** Guiding thresholds for quantum error correction using Lindblad-type noise models.

Articles of interest

These articles describe the Quantum system as outlined in this article.

Open Quantum System Approaches to Superconducting Qubits (https://arxiv.org/html/2402.19241v1)

Quantum Computer Operating System: The Key to Quantum Power (https://www.spinquanta.com/news-detail/quantum-computer-operating-system-the-key-to-quantum-power20250116104617)

Building Quantum Computers A Practical Introduction (https://www.cambridge.org/highereducation/books/building-quantum-computers/6A73C509D3E0F5F0A566A11F6A566A90#overview)

OpenQASM: The Quantum Programming Language. Assembly Programming for Quantum Computers (h

302.02953)
Time Evolution in Open Quantum Systems (https://link.springer.com/chapter/10.1007/978-3-642-23354-8_3)

Challenges

- The Lindblad approach assumes memoryless noise and may not capture non-Markovian dynamics in advanced devices.
- Redfield and other non-Markovian models can describe richer environments but are computationally expensive and sometimes unphysical.
- Hybrid approaches combining Lindblad and non-Markovian models are under investigation.
- Active error suppression techniques (e.g. dynamical decoupling, error mitigation) complement open-system modeling.

Further reading

- Weiss, Ulrich (2012). *Quantum Dissipative Systems*. World Scientific. ISBN 978-9814374910.

Suggested exercises

- Derive the Lindblad master equation for a two-level system coupled to a thermal bath.
- Compare Redfield and Lindblad dynamics for weak system–environment coupling.
- Simulate decoherence of a qubit under amplitude damping using Kraus operators.

This page emphasizes conceptual and mathematical structure rather than step-by-step instruction.

See also

- Quantum
- Quantum A Matter Of Size
- Quantum A Spooky Action at a Distance
- Quantum: A Walk Through the Universe
- Number of independent spatial modes in a spherical volume
- Quantum Computing Algorithms in the NISQ Era
- Quantum Formulas Collection
- Quantum Matter Elements and Particles
- Quantum mechanics

- Quantum optics beam splitter experiments
- Quantum: The Secret of Cohesion: How Waves Hold Matter Together
- Quantum Ultra fast lasers
- Template:Quantum optics operators
- Physical Sciences

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