Quantum mechanics measurements

←Quantum

Attribution: this resource was created by Harold Foppele.

Subject classification: this is a physics resource.

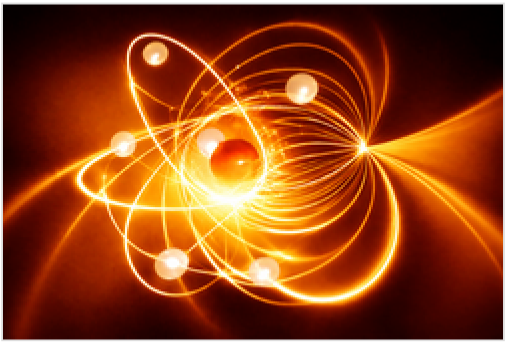
Type classification: this is a quiz resource.

Type classification: this resource is a learning project.

Introduction

Measurement in quantum physics is the manipulation or testing of a physical system to generate a numerical result. Quantum theory's foundational feature is that the outcome it predicts is probabilistic.

In quantum mechanics, calculating the likelihood of an outcome involves linking the mathematical description of a system, its quantum state, with a representation of the measurement . This procedure is formalized by the Born rule. An electron: its quantum state assigns a complex value, or a probability amplitude. The Born rule converts these amplitudes into the chances of detecting the electron in a particular location during a measurement. Quantum theory does not provide certainty, it only provides probabilities. The same quantum state can also be used to predict other properties, such as momentum, if that observable is measured instead. The uncertainty principle makes that precise knowledge of one quantity, like position, comes at the expense of unpredictability in its complementary quantity, like momentum. The fact that experiments violate Bell inequalities shows that this probabilistic behavior cannot be explained by local hidden variables. Randomness is a fundamental feature.



Artistic impression of an atom 6

Common experimental measuring devices

Observable	Typical Measurement Device
Position	Photodetectors, CCD cameras, scanning tunneling microscopes (STM)
Momentum	Time-of-flight detectors, Doppler measurement setups
Spin	Stern–Gerlach apparatus, spin-resolving detectors
Energy / Spectra	Spectrometers, calorimeters, photomultiplier tubes
Photon number / field observables	Photodiodes, homodyne detectors, superconducting qubits readout

quantum states evolve was developed throughout the 20th century, drawing on the methods of linear algebra and functional analysis. Quantum theory has demonstrated experimental accuracy and a range of applications.

At a more philosophical level, discussions continue concerning the interpretation of what measurement means within the theory. Competing interpretations of quantum mechanics offer different resolutions to the so-called measurement problem, the question of how and when quantum possibilities give definite outcomes.

Axiomatic formalism

"Observables" represented by self-adjoint operators

For more coverage of this topic see

- Obervable (quantum mechanics)
- Canonical quantization
- Dirac–von Neumann axioms

In quantum mechanics, physical systems are described by a Hilbert space, where elements correspond to the system's quantum states. The mathematical framework developed by John von Neumann, learns that a measurement is represented by a self-adjoint operator acting on this space, known as an "observable".[1] Operators correspond to the measurable quantities from classical physics such as position, momentum, energy, and angular momentum.

The dimensionality of a system's Hilbert space depends of the system being modeled. For example, the Hilbert space describing a particle with a continuous degree of freedom, like position along a line, is infinite-dimensional and consists of square-integrable functions. In contrast, systems characterized by discrete properties, such as spin, are associated with finite-dimensional Hilbert spaces. Because the mathematics of the finite-dimensional case is considerably simpler, it is often mentioned in pedagogical treatments.

Introductory presentations of quantum mechanics often omit the advanced mathematical subtleties that arise for continuous observables and infinite-dimensional spaces—issues such as bounded versus unbounded operators, convergence of sequences, or unusual spectra like Cantor sets. These complications are rigorously addressed through spectral theory, but detailed discussions are usually reserved for advanced texts. In quantum mechanics, each physical system is associated with a Hilbert space, whose elements represent possible states. In the framework codified by John von Neumann, a measurement is represented by a self-adjoint operator on this space, called an "observable".[2] Observables correspond to familiar classical quantities such as position, momentum, energy, and angular momentum. The dimension of the Hilbert space may be infinite, as for the space of square-integrable functions describing a continuous degree of freedom, or finite, as for spin degrees of freedom. Many treatments focus on the finite-dimensional case, where the mathematics is simpler. Introductory texts often gloss over technical

Measurement based on state projection

See also Projection-valued measure

The eigenvectors of a von Neumann observable form an orthonormal basis for the Hilbert space, and each possible outcome of that measurement corresponds to one of the vectors comprising the basis. A density operator is a positive-semidefinite operator on the Hilbert space whose trace is equal to 1.[2][3] For each measurement that can be defined, the probability distribution over the outcomes of that measurement can be computed from the density operator. The procedure for doing so is the Born rule, which states that

$$P(x_i) = \operatorname{tr}(\Pi_i \rho),$$

where ρ is the density operator, and Π_i is the projection operator onto the basis vector corresponding to the measurement outcome x_i. The average of the eigenvalues of a von Neumann observable, weighted by the Born rule probabilities, is the expectation value of that observable. For an observable A, the expectation value given a quantum state ρ is

$$\langle A \rangle = \operatorname{tr}(A\rho).$$

A density operator that is a rank-1 projection is known as a *pure* quantum state, and all quantum states that are not pure are designated *mixed*. Pure states are also known as *wavefunctions*. Assigning a pure state to a quantum system implies certainty about the outcome of some measurement on that system (i.e., $P(x) = 1$ for some outcome x). Any mixed state can be written as a convex combination of pure states, though not in a unique way.[5] The state space of a quantum system is the set of all states, pure and mixed, that can be assigned to it.

The Born rule associates a probability with each unit vector in the Hilbert space, in such a way that these probabilities sum to 1 for any set of unit vectors comprising an orthonormal basis. Moreover, the probability associated with a unit vector is a function of the density operator and the unit vector, and not of additional information like a choice of basis for that vector to be embedded in. Gleason's theorem establishes the converse: all assignments of probabilities to unit vectors (or, equivalently, to the operators that project onto them) that satisfy these conditions take the form of applying the Born rule to some density operator.[6][7][8]

Positive Operator-Valued Measure (POVM)

See PQVM

A generalized measurement, or positive operator-valued measure (POVM), extends the standard projective measurement framework in quantum mechanics.

$p(i) = \mathrm{Tr}(F_i \rho)$, and after the measurement, the state collapses to $\rho' = \frac{}{p(i)}$.

In contrast, a POVM consists of a set of positive semi-definite operators E_i that satisfy $E_i \geq 0$ and $\sum_i E_i = I$. The probability of obtaining outcome i is given by $p(i) = \mathrm{Tr}(E_i \rho)$. The post-measurement state depends on the specific measurement implementation and is not uniquely determined by the POVM elements.

POVMs provide a more general description of measurements, encompassing cases such as noisy or coarse-grained detectors and indirect measurements involving ancillary systems. Every projective measurement is a special case of a POVM, but not all POVMs correspond to projective measurements on the original system alone. According to Naimark's theorem, any POVM can be realized as a projective measurement on an extended Hilbert space that includes an auxiliary system. In functional analysis and quantum measurement theory, a positive-operator-valued measure (POVM) is a measure whose values are positive semi-definite operators on a Hilbert space. POVMs are a generalisation of projection-valued measures (PVMs) and, correspondingly, quantum measurements described by POVMs are a generalisation of quantum measurement|projection-valued measure described by PVMs. In rough analogy, a POVM is to a PVM what a mixed state is to a pure state. Mixed states are needed to specify the state of a subsystem of a larger system (see Schrödinger–HJW theorem); analogously, POVMs are necessary to describe the effect on a subsystem of a projective measurement performed on a larger system. POVMs are the most general kind of measurement in quantum mechanics, and can also be used in quantum field theory.[9] They are extensively used in the field of quantum information.

In the simplest case, of a POVM with a finite number of elements acting on a finite-dimensional Hilbert space, a POVM is a set of positive semi-definite, matrices $\{F_i\}$ on a Hilbert space $\mathcal{H}$ that sum to the identity matrix,[10]:90

$$\sum_{i=1}^{n} F_i = \mathrm{I}.$$

In quantum mechanics, the POVM element F_i is associated with the measurement outcome i, such that the probability of obtaining it when making a measurement on the quantum state ρ is given by

$$\mathrm{Prob}(i) = \mathrm{tr}(\rho F_i),$$

where tr is the trace operator. When the quantum state being measured is a pure state $|\psi\rangle$ this formula reduces to

$$\mathrm{Prob}(i) = \mathrm{tr}(|\psi\rangle\langle\psi| F_i) = \langle\psi|F_i|\psi\rangle.$$

State change due to measurement

- Main resource Quantum operation

into a product:

$$E_i = A_i^\dagger A_i.$$

The Kraus operators A_i, named for Karl Kraus, provide a specification of the state-change process.[a] They are not necessarily self-adjoint, but the products $A_i^\dagger A_i$ are. If upon performing the measurement the outcome E_i is obtained, then the initial state ρ is updated to

$$\rho \to \rho' = \frac{A_i \rho A_i^\dagger}{\mathrm{Prob}(i)} = \frac{A_i \rho A_i^\dagger}{\mathrm{tr}(\rho E_i)}.$$

An important special case is the Lüders rule, named for Gerhart Lüders.[17][18] If the POVM is itself a PVM, then the Kraus operators can be taken to be the projectors onto the eigenspaces of the von Neumann observable:

$$\rho \to \rho' = \frac{\Pi_i \rho \Pi_i}{\mathrm{tr}(\rho \Pi_i)}.$$

If the initial state ρ is pure, and the projectors Π_i have rank 1, they can be written as projectors onto the vectors $|\psi\rangle$ and $|i\rangle$, respectively. The formula simplifies thus to

$$\rho = |\psi\rangle\langle\psi| \to \rho' = \frac{|i\rangle\langle i|\psi\rangle\langle\psi|i\rangle\langle i|}{|\langle i|\psi\rangle|^2} = |i\rangle\langle i|.$$

Lüders rule has historically been known as the "reduction of the wave packet" or the "collapse of the wavefunction".[18][19][20] The pure state $|i\rangle$ implies a probability-one prediction for any von Neumann observable that has $|i\rangle$ as an eigenvector. Introductory texts on quantum theory often express this by saying that if a quantum measurement is repeated in quick succession, the same outcome will occur both times. This is an oversimplification, since the physical implementation of a quantum measurement may involve a process like the absorption of a photon; after the measurement, the photon does not exist to be measured again.[10]:91

We can define a linear, trace-preserving, completely positive map, by summing over all the possible post-measurement states of a POVM without the normalisation:

$$\rho \to \sum_i A_i \rho A_i^\dagger.$$

It is an example of a quantum channel,[11]:150 and can be interpreted as expressing how a quantum state changes if a measurement is performed but the result of that measurement is lost.[11]:159

dimensional. A pure state for a qubit can be written as a linear combination of two orthogonal basis states $|0\rangle$ and $|1\rangle$ with complex coefficients:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

A measurement in the $(|0\rangle, |1\rangle)$ basis will yield outcome $|0\rangle$ with probability $|\alpha|^2$ and outcome $|1\rangle$ with probability $|\beta|^2$, so by normalization,

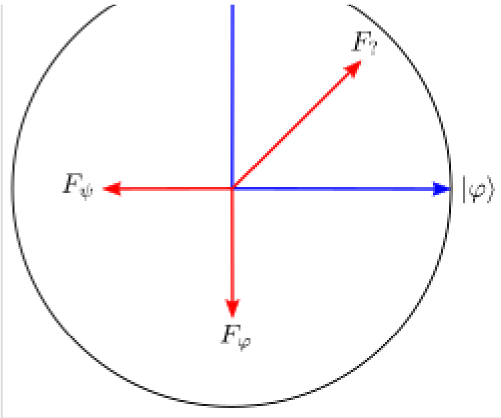
$$|\alpha|^2 + |\beta|^2 = 1.$$

An arbitrary state for a qubit can be written as a linear combination of the Pauli matrices, which provide a basis for 2×2 self-adjoint matrices:[11]:126

$$\rho = \tfrac{1}{2}\left(I + r_x\sigma_x + r_y\sigma_y + r_z\sigma_z\right),$$

where the real numbers (r_x, r_y, r_z) are the coordinates of a point within the unit ball and

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$



Bloch sphere representation of states (in blue) and optimal POVM (in red) for unambiguous quantum state discrimination[21] on the states $|\psi\rangle = |0\rangle$ and $|\varphi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. Note that on the Bloch sphere orthogonal states are antiparallel.

POVM elements can be represented likewise, though the trace of a POVM element is not fixed to equal 1. The Pauli matrices are traceless and orthogonal to one another with respect to the Hilbert–Schmidt inner product, and so the coordinates (r_x, r_y, r_z) of the state ρ are the expectation values of the three von Neumann measurements defined by the Pauli matrices.[11]:126 If such a measurement is applied to a qubit, then by the Lüders rule, the state will update to the eigenvector of that Pauli matrix corresponding to the measurement outcome. The eigenvectors of σ_z are the basis states $|0\rangle$ and $|1\rangle$, and a measurement of σ_z is often called a measurement in the "computational basis."[11]:76 After a measurement in the computational basis, the outcome of a σ_x or σ_y measurement is maximally uncertain.

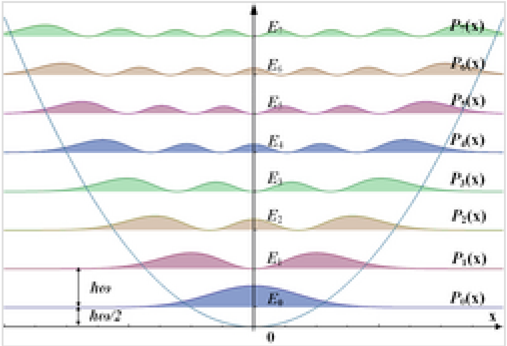
A pair of qubits together form a system whose Hilbert space is 4-dimensional. One significant von Neumann measurement on this system is that defined by the Bell basis,[22]:36 a set of four maximally entangled states:

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)$$

A common and useful example of quantum mechanics applied to a continuous degree of freedom is the quantum harmonic oscillator.[23]:24 This system is defined by the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2,$$



Probability density $P_n(x)$ for the outcome of a position measurement given the energy eigenstate $|n\rangle$ of a 1D harmonic oscillator

where H, the momentum operator p and the position operator x are self-adjoint operators on the Hilbert space of square-integrable functions on the real line. The energy eigenstates solve the time-independent Schrödinger equation:

$$H|n\rangle = E_n|n\rangle.$$

These eigenvalues can be shown to be given by

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right),$$

and these values give the possible numerical outcomes of an energy measurement upon the oscillator. The set of possible outcomes of a *position* measurement on a harmonic oscillator is continuous, and so predictions are stated in terms of a probability density function $P(x)$ that gives the probability of the measurement outcome lying in the infinitesimal interval from x to $x + dx$.

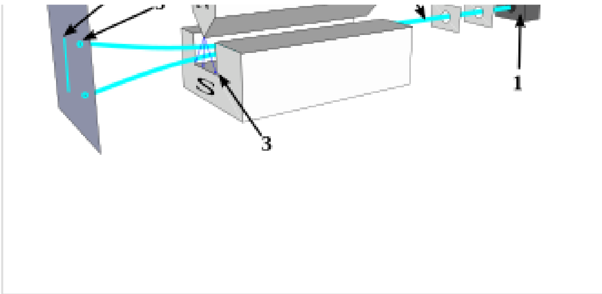
History of the measurement concept

The "old quantum theory"

Main resource: Old quantum theory

The old quantum theory is a collection of results from the years 1900–1925[24] which predate modern quantum mechanics. The theory was never complete or self-consistent, but was rather a set of heuristic corrections to classical mechanics.[25] The theory is now understood as a semi-classical approximation[26] to modern quantum mechanics.[27] Notable results from this period include Max Planck's calculation of the blackbody radiation spectrum, Albert Einstein's explanation of the photoelectric effect, Einstein and Peter Debye's work on the specific heat of solids, Niels Bohr and |Hendrika van Leeuwen's proof that classical physics cannot account for magnetism, Bohr's model of the hydrogen atom and ArnoWld Sommerfeld's extension of the Bohr model to include relativistic effects.

original experiment, silver atoms were sent through a spatially varying magnetic field, which deflected them before they struck a detector screen, such as a glass slide. Particles with non-zero magnetic moment are deflected, due to the magnetic field gradient, from a straight path. The screen reveals discrete points of accumulation, rather than a continuous distribution, owing to the particles' quantized spin.[31][32][33]



Stern–Gerlach experiment: Silver atoms travelling through an inhomogeneous magnetic field, and being deflected up or down depending on their spin; (1) furnace, (2) beam of silver atoms, (3) inhomogeneous magnetic field, (4) classically expected result, (5) observed result.

Transition to the "new" quantum theory

A 1925 paper by Werner Heisenberg, known in English as "Quantum theoretical re-interpretation of kinematic and mechanical relations", marked a pivotal moment in the maturation of quantum physics.[34] Heisenberg sought to develop a theory of atomic phenomena that relied only on "observable" quantities. At the time, and in contrast with the later standard presentation of quantum mechanics, Heisenberg did not regard the position of an electron bound within an atom as "observable". Instead, his principal quantities of interest were the frequencies of light emitted or absorbed by atoms.[34]

The uncertainty principle dates to this period. It is frequently attributed to Heisenberg, who introduced the concept in analyzing a thought experiment where one attempts to measure an electron's position and momentum simultaneously. However, Heisenberg did not give precise mathematical definitions of what the "uncertainty" in these measurements meant. The precise mathematical statement of the position-momentum uncertainty principle is due to Earle Hesse Kennard, Wolfgang Pauli, and Hermann Weyl, and its generalization to arbitrary pairs of noncommuting observables is due to Howard P. Robertson and Erwin Schrödinger.[35][36]

Writing x and p for the self-adjoint operators representing position and momentum respectively, a standard deviation of position can be defined as

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2},$$

and likewise for the momentum:

$$\sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}.$$

The Kennard–Pauli–Weyl uncertainty relation is

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}.$$

commutator of these two operators is

$$[A,B]=AB-BA,$$

and this provides the lower bound on the product of standard deviations:

$$\sigma_A\sigma_B \geq \left|\frac{1}{2i}\langle[A,B]\rangle\right| = \frac{1}{2}\left|\langle[A,B]\rangle\right|.$$

Substituting in the canonical commutation relation $[x,p]=i\hbar$, an expression first postulated by Max Born in 1925,[38] recovers the Kennard–Pauli–Weyl statement of the uncertainty principle.

From uncertainty to no-hidden-variables

Main resources: EPR paradox, Bell's theorem, and Bell test

The existence of the uncertainty principle naturally raises the question of whether quantum mechanics can be understood as an approximation to a more exact theory. Do there exist "hidden variables", more fundamental than the quantities addressed in quantum theory itself, knowledge of which would allow more exact predictions than quantum theory can provide? A collection of results, most significantly Bell's theorem, have demonstrated that broad classes of such hidden-variable theories are in fact incompatible with quantum physics.

John Stewart Bell published the theore|John Stewart Bellm now known by his name in 1964, investigating more deeply a thought experiment originally proposed in 1935 by Einstein, Boris Podolsky and Nathan Rosen.[39][40] According to Bell's theorem, if nature actually operates in accord with any theory of *local* hidden variables, then the results of a Bell test will be constrained in a particular, quantifiable way. If a Bell test is performed in a laboratory and the results are *not* thus constrained, then they are inconsistent with the hypothesis that local hidden variables exist. Such results would support the position that there is no way to explain the phenomena of quantum mechanics in terms of a more fundamental description of nature that is more in line with the rules of classical physics. Many types of Bell test have been performed in physics laboratories, often with the goal of ameliorating problems of experimental design or set-up that could in principle affect the validity of the findings of earlier Bell tests. This is known as "closing loopholes in Bell tests". To date, Bell tests have found that the hypothesis of local hidden variables is inconsistent with the way that physical systems behave.[41][42]

Quantum systems as measuring devices

The Robertson–Schrödinger uncertainty principle establishes that when two observables do not commute, there is a tradeoff in predictability between them. The Wigner–Araki–Yanase theorem demonstrates another consequence of non-commutativity: the presence of a conservation law limits the accuracy with which observables that fail to commute with the conserved quantity can be measured.[43] Further investigation in this line led to the formulation of the Wigner–Yanase skew information.[44]

of Maxwell's equations. But the devices used to build the experimental apparatus are themselves physical systems, and so quantum mechanics should be applicable to them as well. Beginning in the 1950s, Léon Rosenfeld, Carl Friedrich von Weizsäcker and others tried to develop consistency conditions that expressed when a quantum-mechanical system could be treated as a measuring apparatus.[45] One proposal for a criterion regarding when a system used as part of a measuring device can be modeled semiclassically relies on the Wigner function, a quasiprobability distribution that can be treated as a probability distribution on phase space in those cases where it is everywhere non-negative.[3]:375

Decoherence

Main resource: Quantum decoherence

A quantum state for an imperfectly isolated system will generally evolve to be entangled with the quantum state for the environment. Consequently, even if the system's initial state is pure, the state at a later time, found by taking the partial trace of the joint system-environment state, will be mixed. This phenomenon of entanglement produced by system-environment interactions tends to obscure the more exotic features of quantum mechanics that the system could in principle manifest. Quantum decoherence, as this effect is known, was first studied in detail during the 1970s.[46] (Earlier investigations into how classical physics might be obtained as a limit of quantum mechanics had explored the subject of imperfectly isolated systems, but the role of entanglement was not fully appreciated.[45]) A significant portion of the effort involved in quantum computing research is to avoid the deleterious effects of decoherence.[47][22]:239

To illustrate, let ρ_S denote the initial state of the system, ρ_E the initial state of the environment and H the Hamiltonian specifying the system-environment interaction. The density operator ρ_E can be diagonalized and written as a linear combination of the projectors onto its eigenvectors:

$$\rho_E = \sum_i p_i |\psi_i\rangle\langle\psi_i|.$$

Expressing time evolution for a duration t by the unitary operator $U = e^{-iHt/\hbar}$, the state for the system after this evolution is

$$\rho_S' = \mathrm{tr}_E U \left[\rho_S \otimes \left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right)\right] U^\dagger,$$

which evaluates to

$$\rho_S' = \sum_{ij} \sqrt{p_i} \langle\psi_j|U|\psi_i\rangle \rho_S \sqrt{p_i} \langle\psi_i|U^\dagger|\psi_j\rangle.$$

The quantities surrounding ρ_S can be identified as Kraus operators, and so this defines a quantum channel.[46]

Hilbert space.

Quantum information and computation

Quantum information science studies how information science and its application as technology depend on quantum-mechanical phenomena. Understanding measurement in quantum physics is important for this field in many ways, some of which are briefly surveyed here.

Measurement, entropy, and distinguishability

The von Neumann entropy is a measure of the statistical uncertainty represented by a quantum state. For a density matrix ρ, the von Neumann entropy is

$$S(\rho) = -\mathrm{tr}(\rho \log \rho);$$

writing ρ in terms of its basis of eigenvectors,

$$\rho = \sum_i \lambda_i |i\rangle\langle i|,$$

the von Neumann entropy is

$$S(\rho) = -\sum_i \lambda_i \log \lambda_i .$$

This is the Shannon entropy of the set of eigenvalues interpreted as a probability distribution, and so the von Neumann entropy is the Shannon entropy of the random variable defined by measuring in the eigenbasis of ρ. Consequently, the von Neumann entropy vanishes when ρ is pure.[11]:320 The von Neumann entropy of ρ can equivalently be characterized as the minimum Shannon entropy for a measurement given the quantum state ρ, with the minimization over all POVMs with rank-1 elements.[11]:323

Many other quantities used in quantum information theory also find motivation and justification in terms of measurements. For example, the trace distance between quantum states is equal to the largest *difference in probability* that those two quantum states can imply for a measurement outcome:[11]:254

$$\frac{1}{2}||\rho - \sigma|| = \max_{0 \leq E \leq I} [\mathrm{tr}(E\rho) - \mathrm{tr}(E\sigma)].$$

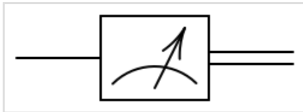
Similarly, the fidelity of two quantum states, defined by

$$F(\rho, \sigma) = \left(\mathrm{Tr} \sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2,$$

$$1-\sqrt{F(\rho,\sigma)} \leq \frac{1}{2}||\rho-\sigma|| \leq \sqrt{1-F(\rho,\sigma)}.$$

Quantum circuits

Main resource: Quantum circuit



Circuit representation of measurement. The single line on the left-hand side stands for a qubit, while the two lines on the right-hand side represent a classical bit.

Quantum circuits are a model for quantum computation in which a computation is a sequence of quantum gates followed by measurements.[22]:93 The gates are reversible transformations on a quantum mechanical analog of an *n*-bit register. This analogous structure is referred to as an *n*-qubit register. Measurements, drawn on a circuit diagram as stylized pointer dials, indicate where and how a result is obtained from the quantum computer after the steps of the computation are executed. Without loss of generality, one can work with the standard circuit model, in which the set of gates are single-qubit unitary transformations and controlled NOT gates on pairs of qubits, and all measurements are in the computational basis.[22]:93[48]

Measurement-based quantum computation

Main resource: One-way quantum computer

Measurement-based quantum computation (MBQC) is a model of quantum computing in which the answer to a question is, informally speaking, created in the act of measuring the physical system that serves as the computer.[22]:317[49][50]

Quantum tomography

Main resource: Tomography

Quantum state tomography is a process by which, given a set of data representing the results of quantum measurements, a quantum state consistent with those measurement results is computed.[51] It is named by analogy with tomography, the reconstruction of three-dimensional images from slices taken through them, as in a CT scan. Tomography of quantum states can be extended to tomography of quantum channels[51] and even of measurements.[52]

Quantum metrology

Main resource: Quantum metrology

Quantum metrology is the use of quantum physics to aid the measurement of quantities that, generally, had meaning in classical physics, such as exploiting quantum effects to increase the precision with which a length can be measured.[53] A celebrated example is the introduction of squeezed light into the LIGO experiment, which increased its sensitivity to gravitational waves.[54][55]

The range of physical procedures to which the mathematics of quantum measurement can be applied is very broad.[56] In the early years of the subject, laboratory procedures involved the recording of spectral lines, the darkening of photographic film, the observation of scintillations, finding tracks in cloud chambers, and hearing clicks from Geiger counters.[b] Language from this era persists, such as the description of measurement outcomes in the abstract as "detector clicks".[58]

The double-slit experiment is a prototypical illustration of quantum interference, typically described using electrons or photons. The first interference experiment to be carried out in a regime where both wave-like and particle-like aspects of photon behavior are significant was G. I. Taylor's test in 1909. Taylor used screens of smoked glass to attenuate the light passing through his apparatus, to the extent that, in modern language, only one photon would be illuminating the interferometer slits at a time. He recorded the interference patterns on photographic plates; for the dimmest light, the exposure time required was roughly three months.[59][60] In 1974, the Italian physicists Pier Giorgio Merli, Gian Franco Missiroli, and Giulio Pozzi implemented the double-slit experiment using single electrons and a television tube (CRT).[61] A quarter-century later, a team at the University of Vienna performed an interference experiment with buckyballs, in which the buckyballs that passed through the interferometer were ionized by a laser, and the ions then induced the emission of electrons, emissions which were in turn amplified and detected by an electron multiplier.[62]

Modern quantum optics experiments can employ single-photon detectors. For example, in the "BIG Bell test" of 2018, several of the laboratory setups used single-photon avalanche diodes. Another laboratory setup used superconducting qubits.[41] The standard method for performing measurements upon superconducting qubits is to couple a qubit with a resonator in such a way that the characteristic frequency of the resonator shifts according to the state for the qubit, and detecting this shift by observing how the resonator reacts to a probe signal.[63]

Interpretations of quantum mechanics

Main resource: Interpretations of quantum mechanics

Despite the consensus among scientists that quantum physics is in practice a successful theory, disagreements persist on a more philosophical level. Many debates in the area known as quantum foundations concern the role of measurement in quantum mechanics. Recurring questions include which interpretation of probability theory is best suited for the probabilities calculated from the Born rule; and whether the apparent randomness of quantum measurement outcomes is fundamental, or a consequence of a deeper deterministic process.[64][65][66] Worldviews that present answers to questions like these are known as "interpretations" of quantum mechanics; as the physicist N. David Mermin once quipped, "New interpretations appear every year. None ever disappear."[67]

A central concern within quantum foundations is the "quantum measurement problem," though how this problem is delimited, and whether it should be counted as one question or multiple separate issues, are contested topics.[57][68] Of primary interest is the seeming disparity between apparently distinct types of time evolution. Von Neumann declared that quantum mechanics contains "two fundamentally different former is stochastic and discontinuous, writes von Neumann, and the latter deterministic and continuous. This dichotomy has set the tone for much later debate.[70][71] Some interpretations of quantum mechanics find the reliance upon two different types of time evolution distasteful and regard the ambiguity of when to invoke one or the other as a deficiency of the way quantum theory was historically presented.[72] To bolster these interpretations, their proponents have worked to derive ways of regarding "measurement" as a secondary concept and deducing the seemingly stochastic effect of measurement processes as approximations to more fundamental deterministic dynamics. However, consensus has not been achieved among proponents of the correct way to implement this program, and in particular how to justify the use of the Born rule to calculate probabilities.[73][74] Other interpretations regard quantum states as statistical information about quantum systems, thus asserting that abrupt and discontinuous changes of quantum states are not problematic, simply reflecting updates of the available information.[56][75] Of this line of thought, Bell asked, "*Whose* information? Information about *what*?"[72] Answers to these questions vary among proponents of the informationally-oriented interpretations.[65][75]

Niels Bohr and Albert Einstein, pictured here at Paul Ehrenfest's home in Leiden (December 1925), had a long-running collegial dispute about what quantum mechanics implied for the nature of reality.

See also

- Einstein's thought experiments
- Holevo's theorem
- Quantum error correction
- Quantum limit
- Quantum logic
- Quantum Zeno effect
- Schrödinger's cat
- SIC-POVM

Further reading

Search Wikiquote for quotations related to: ***Quantum mechanics measurements***

- *Quantum Theory and Measurement*. Princeton University Press. 1983. ISBN 978-0-691-08316-2.
- Braginsky, Vladimir B.; Khalili, Farid Ya. (1992). *Quantum Measurement*. Cambridge University Press. ISBN 978-0-521-41928-4.
- Greenstein, George S.; Zajonc, Arthur G. (2006). *The Quantum Challenge: Modern Research On The Foundations Of Quantum Mechanics* (2nd ed.). ISBN 978-0-7637-2470-2.
- Alter, Orly; Yamamoto, Yoshihisa (2001). *Quantum Measurement of a Single System*. New York: Wiley. doi:10.1002/9783527617128. ISBN 978-0-471-28308-9.
- Andrew N. Jordan; Irfan A. Siddiqi (2024). *Quantum Measurement: Theory and Practice*. Cambridge University Press. ISBN 978-1-009-10006-9.

a. Hellwig and Kraus[illegible] originally introduced operators with two indices, A_{ij}, such that $\sum_j A_{ij} A_{ij}^\dagger = E_i$. The extra index does not affect the computation of the measurement outcome probability, but it does play a role in the state-update rule, with the post-measurement state being now proportional to $\sum_j A_{ij}^\dagger \rho A_{ij}$. This can be regarded as representing E_i as a coarse-graining together of multiple outcomes of a more fine-grained POVM.[14][15][16] Kraus operators with two indices also occur in generalized models of system-environment interaction.[10]:364
b. The glass plates used in the Stern–Gerlach experiment did not darken properly until Stern breathed on them, accidentally exposing them to sulfur from his cheap cigars.[32][57]

See Also

- Quantum
- Quantum A Matter Of Size
- Quantum A Spooky Action at a Distance
- Quantum: A Walk Through the Universe
- Number of independent spatial modes in a spherical volume
- Quantum Computing Algorithms in the NISQ Era
- Quantum Formulas Collection
- Quantum Matter Elements and Particles
- Quantum mechanics
- Quantum mechanics/Timeline
- Quantum mechanics measurements
- Quantum Noisy Qubits
- Quantum optics beam splitter experiments
- Quantum: The Secret of Cohesion: How Waves Hold Matter Together
- Quantum Ultra fast lasers
- Template:Quantum optics operators
- Physical Sciences

Quantum mechanics	Search for *Quantum mechanics measurements* on Wikipedia.
Background	Introduction · History (Timeline) · Classical mechanics · Old quantum theory · Glossary
Fundamentals	Born rule · Bra–ket notation · Complementarity · Density matrix · Energy level (Ground state · Excited state · Degenerate levels · Zero-point energy) · Entanglement · Hamiltonian · Interference · Decoherence · Measurement · Nonlocality · Quantum state (quantum jump) · Superposition · Tunnelling · Scattering theory ·

	Path integral formulation · Phase space
Equations	Klein–Gordon · Dirac · Weyl · Majorana · Rarita–Schwinger · Pauli · Rydberg · Schrödinger
Interpretations	Bayesian · Consciousness causes collapse · Consistent histories · Copenhagen · de Broglie–Bohm · Ensemble · Hidden-variable (Local (Superdeterminism)) · Many-worlds · Objective collapse · Quantum logic · Relational · Transactional
Experiments	Bell test · Davisson–Germer · Delayed-choice quantum eraser · Double-slit · Franck–Hertz · Mach–Zehnder interferometer · Elitzur–Vaidman · Popper · Quantum eraser · Stern–Gerlach · Wheeler's delayed choice
Science	Quantum biology · Quantum chemistry · Quantum chaos · Quantum cosmology · Quantum differential calculus · Quantum dynamics · Quantum geometry · Quantum measurement problem · Quantum mind · Quantum stochastic calculus · Quantum spacetime
Technology	Quantum algorithms · Quantum amplifier · Quantum bus · Quantum cellular automata (Quantum finite automata) · Quantum channel · Quantum circuit · Quantum complexity theory · Quantum computing (Timeline) · Quantum cryptography · Quantum electronics · Quantum error correction · Quantum imaging · Quantum image processing · Quantum information · Quantum key distribution · Quantum logic · Quantum logic gates · Quantum machine · Quantum machine learning · Quantum metamaterial · Quantum metrology · Quantum network · Quantum neural network · Quantum optics · Quantum programming · Quantum sensing · Quantum simulator · Quantum teleportation
Extensions	Quantum fluctuation · Casimir effect · Quantum statistical mechanics · Quantum field theory (History) · Quantum gravity · Relativistic quantum mechanics
Related	Schrödinger's cat (in popular culture) · Wigner's friend · EPR paradox · Quantum mysticism
Category:Quantum mechanics	

Documentation

(2001). *Statistical Structure of Quantum Theory*. Lecture Notes in Physics. Springer. ISBN 3-540-42082-7. OCLC 318268606.

2. Holevo, Alexander S. (2001). *Statistical Structure of Quantum Theory*. Lecture Notes in Physics. Springer. ISBN 3-540-42082-7. OCLC 318268606.
3. Peres, Asher (1995). *Quantum Theory: Concepts and Methods*. Kluwer Academic Publishers. ISBN 0-7923-2549-4.
4. Tao, Terry (12 August 2014). "Avila, Bhargava, Hairer, Mirzakhani". *What's New*. Retrieved 9 February 2020.
5. Kirkpatrick, K. A. (February 2006). "The Schrödinger-HJW Theorem". *Foundations of Physics Letters* **19** (1): 95–102. doi:10.1007/s10702-006-1852-1. ISSN 0894-9875.
6. Gleason, Andrew M. (1957). "Measures on the closed subspaces of a Hilbert space". *Indiana University Mathematics Journal* **6** (4): 885–893. doi:10.1512/iumj.1957.6.56050.

"Quantum States and Generalized Observables: A Simple Proof of Gleason's Theorem". *Physical Review Letters* **91** (12). doi:10.1103/PhysRevLett.91.120403. PMID 14525351.

8. Caves, Carlton M.; Fuchs, Christopher A.; Manne, Kiran K.; Renes, Joseph M. (2004). "Gleason-Type Derivations of the Quantum Probability Rule for Generalized Measurements". *Foundations of Physics* **34** (2): 193–209. doi:10.1023/B:FOOP.0000019581.00318.a5.
9. Peres, Asher; Terno, Daniel R. (2004). "Quantum information and relativity theory". *[[W:Reviews of Modern Physics]|Reviews of Modern Physics]]* **76** (1): 93–123. doi:10.1103/RevModPhys.76.93.
10. Nielsen, Michael A.; Chuang, Isaac L. (2000). *Quantum Computation and Quantum Information* (1st ed.). Cambridge: [[W:en:Cambridge University Press]|Cambridge University Press]]. ISBN 978-0-521-63503-5. OCLC 634735192.

Quantum Information Theory (2nd ed.). Cambridge University Press. doi:10.1017/9781316809976.001. ISBN 978-1-107-17616-4. OCLC 973404322.

12. Hellwig, K. -E.; Kraus, K. (September 1969). "Pure operations and measurements". *Communications in Mathematical Physics* **11** (3): 214–220. doi:10.1007/BF01645807. ISSN 0010-3616. https://projecteuclid.org/download/pdf_1/euclid.cmp/1103841220.
13. Kraus, Karl (1983). *States, effects, and operations: fundamental notions of quantum theory*. Lectures in mathematical physics at the University of Texas at Austin. **190**. Springer-Verlag. ISBN 978-3-5401-2732-1. OCLC 925001331. https://books.google.com/books?id=fRBBAQAAIAAJ.
14. Barnum, Howard; Nielsen, M. A.; Schumacher, Benjamin (1 June 1998). "Information transmission through a noisy quantum channel". *Physical Review A* **57** (6): 4153–4175. doi:10.1103/PhysRevA.57.4153. ISSN 1050-2947.

finite-strength quantum measurements". *Physical Review A* **63** (6). doi:10.1103/PhysRevA.63.062305. ISSN 1050-2947.

16. Poulin, David (7 February 2005). "Macroscopic observables". *Physical Review A* **71** (2). doi:10.1103/PhysRevA.71.022102. ISSN 1050-2947.
17. Lüders, Gerhart (1950). "Über die Zustandsänderung durch den Messprozeß". *Annalen der Physik* **443** (5–8): 322. doi:10.1002/andp.19504430510. Translated by K. A. Kirkpatrick as Lüders, Gerhart (3 April 2006). "Concerning the state-change due to the measurement process". *Annalen der Physik* **15** (9): 663–670. doi:10.1002/andp.200610207.
18. Busch, Paul; Lahti, Pekka (2009), Greenberger, Daniel; Hentschel, Klaus; Weinert, Friedel (eds.), "Lüders Rule", *Compendium of Quantum Physics*, Springer Berlin Heidelberg, pp. 356–358, doi:10.1007/978-3-540-70626-7_110, ISBN 978-3-540-70622-9

Physics". In Radnitzky, Gerard; Andersson, Gunnar. *The Structure and Development of Science*. Boston Studies in the Philosophy of Science. **59**. Dordrecht: Springer Netherlands. pp. 41–61. doi:10.1007/978-94-009-9459-1_3. ISBN 978-90-277-0995-0. http://link.springer.com/10.1007/978-94-009-9459-1_3. Retrieved 2024-03-26.

20. Pessoa, Osvaldo (2022). "The Measurement Problem". In Freire, Olival. *The Oxford Handbook of the History of Quantum Interpretations*. Oxford University Press. pp. 281–302. doi:10.1093/oxfordhb/9780198844495.013.0012. ISBN 978-0-191-88008-7.
21. Peres, Asher; Terno, Daniel R. (1998). "Optimal distinction between non-orthogonal quantum states". *Journal of Physics A: Mathematical and General* **31** (34): 7105–7111. doi:10.1088/0305-4470/31/34/013. ISSN 0305-4470.
22. Rieffel, Eleanor G.; Polak, Wolfgang H. (4 March 2011). *Quantum Computing: A Gentle Introduction* (in en). MIT Press. ISBN 978-0-262-01506-6.

Cambridge, United Kingdom: Cambridge University Press. ISBN 978-1-107-11166-0. OCLC 910664598.

24. Pais, Abraham (2005). *Subtle is the Lord: The Science and the Life of Albert Einstein* (illustrated ed.). Oxford University Press. p. 28. ISBN 978-0-19-280672-7. https://books.google.com/books?id=0QYTDAAAQBAJ.
25. ter Haar, D. (1967). *The Old Quantum Theory*. Pergamon Press. pp. 206. ISBN 978-0-08-012101-7. https://archive.org/details/oldquantumtheory0000haar.
26. "Semi-classical approximation". *Encyclopedia of Mathematics*. Retrieved 1 February 2020.
27. Sakurai, J. J.; Napolitano, J. (2014). "Quantum Dynamics". *Modern Quantum Mechanics*. Pearson. ISBN 978-1-292-02410-3. OCLC 929609283.
28. Gerlach, W.; Stern, O. (1922). "Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld". *Zeitschrift für Physik* **9** (1): 349–352. doi:10.1007/BF01326983.

Zeitschrift für Physik **9** (1): 353–355. doi:10.1007/BF01326984.

30. Gerlach, W.; Stern, O. (1922). "Der experimentelle Nachweis des magnetischen Moments des Silberatoms". *Zeitschrift für Physik* **8** (1): 110–111. doi:10.1007/BF01329580. https://zenodo.org/record/1525119.
31. Franklin, Allan; Perovic, Slobodan. "Experiment in Physics, Appendix 5". In Edward N. Zalta. *The Stanford Encyclopedia of Philosophy* (Winter 2016 ed.). https://plato.stanford.edu/archives/win2016/entries/physics-experiment/app5.html. Retrieved 14 August 2018.
32. Friedrich, B.; Herschbach, D. (2003). "Stern and Gerlach: How a Bad Cigar Helped Reorient Atomic Physics". *Physics Today* **56** (12): 53. doi:10.1063/1.1650229.
33. Zhu, Guangtian; Singh, Chandralekha (May 2011). "Improving students' understanding of quantum mechanics via the Stern–Gerlach experiment". *American Journal of Physics* **79** (5): 499–507. doi:10.1119/1.3546093. ISSN 0002-9505. http://aapt.scitation.org/doi/10.1119/1.3546093.

Dover.

35. Busch, Paul; Lahti, Pekka; Werner, Reinhard F. (17 October 2013). "Proof of Heisenberg's Error-Disturbance Relation". *Physical Review Letters* **111** (16). doi:10.1103/PhysRevLett.111.160405. ISSN 0031-9007. PMID 24182239.
36. Appleby, David Marcus (6 May 2016). "Quantum Errors and Disturbances: Response to Busch, Lahti and Werner". *Entropy* **18** (5): 174. doi:10.3390/e18050174.
37. Landau, L.D.; Lifschitz, E.M. (1977). *Quantum Mechanics: Non-Relativistic Theory*. **3** (3rd ed.). Pergamon Press. ISBN 978-0-08-020940-1. OCLC 2284121. https://archive.org/details/QuantumMechanics_104.
38. Born, M.; Jordan, P. (1925). "Zur Quantenmechanik". *Zeitschrift für Physik* **34** (1): 858–888. doi:10.1007/BF01328531.
39. Bell, J. S. (1964). "On the Einstein Podolsky Rosen Paradox". *Physics Physique Физика* **1** (3): 195–200. doi:10.1103/PhysicsPhysiqueFizika.1.195. https://cds.cern.ch/record/111654/files/vol1p195-200_001.pdf.

of Physical Reality be Considered Complete?". *Physical Review* **47** (10): 777–780. doi:10.1103/PhysRev.47.777.

41. The BIG Bell Test Collaboration (9 May 2018). "Challenging local realism with human choices". *Nature* **557** (7704): 212–216. doi:10.1038/s41586-018-0085-3. PMID 29743691.
42. Wolchover, Natalie (7 February 2017). "Experiment Reaffirms Quantum Weirdness". *W:Quanta Magazine*. Retrieved 8 February 2020.

quantenmechanische
r Operatoren", in Mehra, Jagdish (ed.), *Philosophical Reflections and Syntheses*, Springer Berlin Heidelberg, pp. 147–154, doi:10.1007/978-3-642-78374-6_10, ISBN 978-3-540-63372-3

- Araki, Huzihiro; Yanase, Mutsuo M. (15 October 1960). "Measurement of Quantum Mechanical Operators". *Physical Review* **120** (2): 622–626. doi:10.1103/PhysRev.120.622. ISSN 0031-899X.
- Yanase, Mutsuo M. (15 July 1961). "Optimal Measuring Apparatus". *Physical Review* **123** (2): 666–668. doi:10.1103/PhysRev.123.666. ISSN 0031-899X.
- Ahmadi, Mehdi; Jennings, David; Rudolph, Terry (28 January 2013). "The Wigner–Araki–Yanase theorem and the quantum resource theory of asymmetry". *New Journal of Physics* **15** (1). doi:10.1088/1367-2630/15/1/013057. ISSN 1367-2630.

Physical Review Letters **91** (18). doi:10.1103/PhysRevLett.91.180403. PMID 14611271.

45. Camilleri, K.; Schlosshauer, M. (2015). "Niels Bohr as Philosopher of Experiment: Does Decoherence Theory Challenge Bohr's Doctrine of Classical Concepts?". *Studies in History and Philosophy of Modern Physics* **49**: 73–83. doi:10.1016/j.shpsb.2015.01.005.
46. Schlosshauer, M. (2019). "Quantum Decoherence". *Physics Reports* **831**: 1–57. doi:10.1016/j.physrep.2019.10.001.
47. DiVincenzo, David; Terhal, Barbara (March 1998). "Decoherence: the obstacle to quantum computation". *Physics World* **11** (3): 53–58. doi:10.1088/2058-7058/11/3/32. ISSN 0953-8585.
48. Terhal, Barbara M. (7 April 2015). "Quantum error correction for quantum memories". *Reviews of Modern Physics* **87** (2): 307–346. doi:10.1103/RevModPhys.87.307. ISSN 0034-6861.

Quantum Computation on Cluster States". *Physical Review A* **68** (2). doi:10.1103/PhysRevA.68.022312.

50. Childs, Andrew M.; Leung, Debbie W.; Nielsen, Michael A. (17 March 2005). "Unified derivations of measurement-based schemes for quantum computation". *Physical Review A* **71** (3). doi:10.1103/PhysRevA.71.032318. ISSN 1050-2947.
51. Granade, Christopher; Combes, Joshua; Cory, D. G. (1 January 2016). "Practical Bayesian tomography". *New Journal of Physics* **18** (3). doi:10.1088/1367-2630/18/3/033024. ISSN 1367-2630.
52. Lundeen, J. S.; Feito, A.; Coldenstrodt-Ronge, H.; Pregnell, K. L.; Silberhorn, Ch; Ralph, T. C.; Eisert, J.; Plenio, M. B. *et al*. (2009). "Tomography of quantum detectors". *Nature Physics* **5** (1): 27–30. doi:10.1038/nphys1133. ISSN 1745-2481.
53. Braunstein, Samuel L.; Caves, Carlton M. (30 May 1994). "Statistical distance and the geometry of quantum states". *Physical Review Letters* **72** (22): 3439–3443. doi:10.1103/physrevlett.72.3439. PMID 10056200.

Noise Of Empty Space". *Universe Today*. Retrieved 2 February 2020.

55. Ball, Philip (5 December 2019). "Focus: Squeezing More from Gravitational-Wave Detectors". *Physics* **12**: 139. doi:10.1103/Physics.12.139.
56. Peierls, Rudolf (1991). "In defence of "measurement"". *Physics World* **4** (1): 19–21. doi:10.1088/2058-7058/4/1/19. ISSN 2058-7058.
57. Barad, Karen (2007). *Meeting the Universe Halfway: Quantum Physics and the Entanglement of Matter and Meaning* (in en). Duke University Press. ISBN 978-0-8223-3917-5. OCLC 1055296186.
58. Englert, Berthold-Georg (22 November 2013). "On quantum theory". *The European Physical Journal D* **67** (11). doi:10.1140/epjd/e2013-40486-5. ISSN 1434-6079.
59. Taylor, G. I. (1909). "Interference Fringes with Feeble Light". *Mathematical Proceedings of the Cambridge Philosophical Society* **15**: 114. https://archive.org/details/proceedingsofcam15190810camb/page/114/mode/2up. Retrieved 7 December 2024.

Retrieved 24 October 2020.

61. Merli, P G; Missiroli, G F; Pozzi, G (1976). "On the statistical aspect of electron interference phenomena". *American Journal of Physics* **44** (3): 306–307. doi:10.1119/1.10184.
62. Arndt, Markus; Nairz, Olaf; Vos-Andreae, Julian; Keller, Claudia; Van Der Zouw, Gerbrand; Zeilinger, Anton (1999). "Wave–particle duality of C60 molecules". *Nature* **401** (6754): 680–682. doi:10.1038/44348. PMID 18494170.
63. Krantz, Philip; Bengtsson, Andreas; Simoen, Michaël; Gustavsson, Simon; Shumeiko, Vitaly; Oliver, W. D.; Wilson, C. M.; Delsing, Per *et al*. (9 May 2016). "Single-shot read-out of a superconducting qubit using a Josephson parametric oscillator". *Nature Communications* **7** (1). doi:10.1038/ncomms11417. ISSN 2041-1723. PMID 27156732. PMC 4865746. //www.ncbi.nlm.nih.gov/pmc/articles/PMC4865746/.

"A Snapshot of Foundational Attitudes Toward Quantum Mechanics". *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* **44** (3): 222–230. doi:10.1016/j.shpsb.2013.04.004.

65. Cabello, Adán (2017). "Interpretations of quantum theory: A map of madness". In Lombardi, Olimpia; Fortin, Sebastian; Holik, Federico et al.. *What is Quantum Information?*. Cambridge University Press. pp. 138–143. doi:10.1017/9781316494233.009. ISBN 978-1-107-14211-4. Bibcode: 2015arXiv150904711C.
66. Schaffer, Kathryn; Barreto Lemos, Gabriela (24 May 2019). "Obliterating Thingness: An Introduction to the "What" and the "So What" of Quantum Physics". *Foundations of Science* **26**: 7–26. doi:10.1007/s10699-019-09608-5. ISSN 1233-1821.
67. Mermin, N. David (1 July 2012). "Commentary: Quantum mechanics: Fixing the shifty split". *Physics Today* **65** (7): 8–10. doi:10.1063/PT.3.1618. ISSN 0031-9228.

University Press[]].

69. von Neumann, John (2018). Wheeler, Nicholas A.. ed. *Mathematical Foundations of Quantum Mechanics. New Edition*. Princeton University Press. ISBN 9-781-40088-992-1. OCLC 1021172445.
70. Wigner, E. P. (1995), "Review of the Quantum-Mechanical Measurement Problem", in Mehra, Jagdish (ed.), *Philosophical Reflections and Syntheses*, Springer Berlin Heidelberg, pp. 225–244, doi:10.1007/978-3-642-78374-6_19, ISBN 978-3-540-63372-3

Zalta, Edward N.. *Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/entries/qm-copenhagen/.

72. Bell, John (1990). "Against 'measurement'". *Physics World* **3** (8): 33–41. doi:10.1088/2058-7058/3/8/26. ISSN 2058-7058.
73. Kent, Adrian. (2010). "One world versus many: the inadequacy of Everettian accounts of evolution, probability, and scientific confirmation". *Many Worlds?*: 307–354. Oxford University Press.

Edward N.. *Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/entries/qm-everett/.

75. Healey, Richard (2016). "Quantum-Bayesian and Pragmatist Views of Quantum Theory". In Zalta, Edward N.. *Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/entries/quantum-bayesian/.

Retrieved from "https://en.wikiversity.org/w/index.php?title=Quantum_mechanics_measurements&oldid=2794952"